Golven & Optica

4.5

① 1) Intensiteit: W/m2.

$$I = \frac{P}{A}$$

0.5 $$I = \frac{P}{A} = \frac{2000}{10^{-6}\cdot 10^{\overset{4}{-2}}} = 2\cdot 10^{3}\cdot 10^{\overset{10}{\cancel{8}}} = 2{,}000\cdot 10^{\overset{13}{\cancel{11}}}\ \mathrm{W/M^2}.$$

2) $$I = \frac{c\varepsilon_0}{2}E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2\cdot 2{,}000\cdot 10^{11}}{3\cdot 10^{8}\cdot 8{,}854\cdot 10^{-12}}}$$

2 $$= 1{,}2\cdot 10^{7}\ \mathrm{V/M}.$$

2 3) $$E_0 = cB_0 \Rightarrow B_0 = \frac{E_0}{c} = \frac{1{,}2\cdot 10^{7}}{3\cdot 10^{8}} = 0.04\ \frac{\mathrm{V\cdot s}}{\mathrm{M^2}}$$

3.5

② 1) De fysische oorsprong van de fase gegeven door

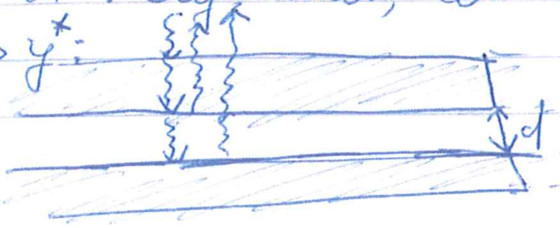
$$\frac{4\pi}{\lambda_f} d\cos\theta_t$$

0.5 is de straal die niet direct reflecteert maar juist de film binnenbringt (transmitted), en dan weerkaatst. De fase op punt A' hangt af van de dikte van de film, $d\cos\theta_t$, en natuurlijk λ_f, vanwege de breking. Dus: $\frac{4\pi}{\lambda_f} d\cos\theta_t$ is fase op A'.

1 is de lichtstraal die direct gereflecteerd wordt. De fase op A is niet 0, want punt A bevindt zich iets verder dan het reflectiepunt. De fase is hier π. (lijn A−A' ⊥ op straal 1)

Het faseverschil δ is dan: $\delta = \frac{4\pi}{\lambda_f} d\cos\theta_t - \pi$.

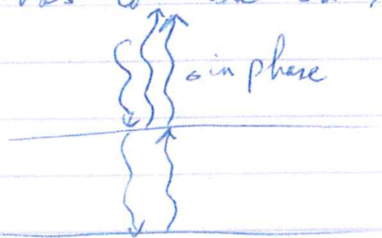
2) If we zoom in very much, we can represent the figure by*:



* Sorry for

aka: Some ~~beams~~ rays get reflected directly, some transmit and then reflect.

In order to get a maximum, the difference in optical Path length (ΔOPL) has to be an multiple of the wavelengthe:

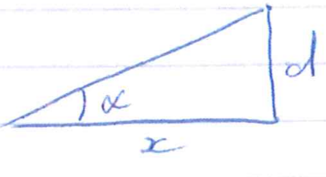


v $\Delta OPL = m\lambda \quad (m=0,1,2,\dots)$

because the transmitted wave travels twice the distance d, we get:

$$2d = m\lambda . \rightarrow d = \frac{m\lambda}{2}$$

Now from mathematics:



3 $d = x \tan \alpha$.

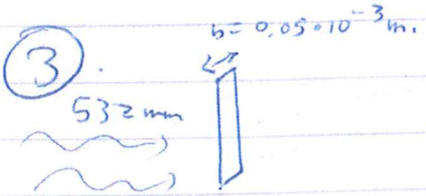
$$\rightarrow x \tan\alpha = \frac{m\lambda}{2}$$

$$\tan\alpha = \frac{m\lambda}{2\cancel{x}} \longrightarrow \alpha = \tan^{-1}\left(\frac{m\lambda}{2x}\right).$$

We want α in terms of neighbouring maxima. Therefore m = 1.

$$\text{So: } \alpha = \tan^{-1}\left(\frac{\lambda}{2x}\right) = \Delta x!$$

③.



Central maximum; width is twice the angular width between the ~~first~~ central maximum and first minimum:

④

$$\sin\beta = 0 \qquad (\beta \neq 0).$$

1 $\frac{kb}{2}\sin(\theta') = \pi, 2\pi, 3\pi \dots \rightarrow$ We only need π. (first minimum)

$$\sin\theta' = \frac{2\pi}{kb}$$

$$\theta' = \sin^{-1}\left(\frac{2\pi}{kb}\right);$$ now $k = \frac{2\pi}{\lambda}$, So:

9. $$\theta' = \sin^{-1}\left(\frac{\lambda}{b}\right)$$

Therefore the angular width of the central maximum is:

$$\theta = 2\sin^{-1}\left(\frac{\lambda}{b}\right) = 2\sin^{-1}\left(\frac{532\cdot10^{-9}}{0.05\cdot10^{-3}}\right)$$

1

$$= 1.22^{0}$$

④ Fraunhofer diffraction occurs when

(5.5)

$R > \frac{a^2}{\lambda}$ with a the radius of the apperture and R ~~the~~ is the least of ρ and r_0.

Now the surface of the aperture is:

$$A = \pi a^2 = \frac{\pi\rho r_0\lambda}{\rho+r_0} \rightarrow a^2 = \frac{\cancel{\pi}\rho r_0\lambda}{\rho+r_0}$$

So $$R \overset{?}{>} \frac{\rho r_0\lambda}{\rho+r_0}\cdot\frac{1}{\lambda} = \frac{\rho r_0}{\rho+r_0}$$

$R =$ ~~ρ~~ or r_0 for instance.

$$r_0 \overset{?}{>} \frac{\rho r_0}{\rho+r_0} \Rightarrow R > \frac{\rho r_0}{\rho+r_0}$$

[redacted]

This is true because R is r_0 or ρ, whichever is the least. [redacted]

[redacted]

to show this, consider: $x \overset{?}{>} \frac{xy}{x+y}$

$$x > \frac{xy}{x+y} \rightarrow \frac{y}{x+y} \overset{?}{\cancel{>}} < 1$$

$$\frac{y}{x+y} \cancel{>} < 1 \text{ if } y>0 \text{ and } x>0.$$

because r_0 and ρ are both >0, (distance $\neq$ negative).

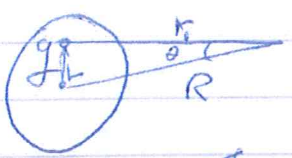
$$R > \frac{\rho r_0}{r_0 + \rho}. \qquad \square$$

⑤ 1) $E(Y, Z) = \frac{\varepsilon_A e^{i(\omega t - kR)}}{R} \iint_{\text{opening}} e^{ik(Y_y + Z_z)/R} \, d_y d_z.$

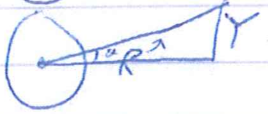
aperture function $A(y, z)$

0.5

$$A = \begin{cases} 1 & \text{if } x, y \text{ (or } r\text{) inside opening} \\ 0 & \text{~~if~~ elsewhere} \end{cases}$$



$\rightarrow \frac{y}{r_1} = \overset{\sin}{\cancel{\cos}}\,\theta \quad r_1 \approx \frac{y}{\theta}.$



$\rightarrow \frac{R}{\cancel{Y}} = \sin\theta \approx \theta \simeq \frac{R}{Y}$

$$r_1 = \frac{yY}{R}$$

The same holds for z: $r_2 = \frac{zZ}{R}.$

Therefore $r = \frac{yY + zZ}{R}$

5.5

⑤ 2) define: $k_y = \frac{k Y}{R}$ Golven & Optica.

1 $k_z = \frac{k Z}{R}.$

Now,

$$E(Y, Z) = \frac{e^{i(\omega t - kR)}}{R} \iint_{\text{opening}} A(y, z)\, e^{i(k_y y + k_z Z)}\, dy\, dz.$$

Because $A(y, z) = 0$ outside the opening, we can just intigrate over all surface:

$$E(Y, Z) = \frac{e^{i(\omega t - kR}}{R} \iint_{-\infty}^{\infty} A(y, z)\, e^{i(k_y y + k_z z)}\, dy\, dz$$

2

$$= \frac{e^{i(\omega t - kR)}}{R} \mathcal{F}(A(y, z))$$

$$= C \cdot \mathcal{F}(A(y, z)) \; ; \; C = \frac{e^{i(\omega t - kR)}}{R}.$$

2 $\Rightarrow E(Y, Z)$ is dependent on ~~as as~~ the Fourier transform of the apperture function!